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COMMENT

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D Stauffer

Institute of Theoretical Physics, Cologne University, 5000 Köln, West Germany and Department of Physics and Astronomy, Tel Aviv University, Tel Aviv, Israel 69978

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Abstract. Computer simulations confirm Dhar's prediction that the asymptotic exponent for the RMS distance against time relation is zero right at the percolation threshold, if a particle diffuses under the influence of a constant force in a random medium. For intermediate times and weak force, the exponent for the mean displacement (as a function of time) in the direction of the force confirms the prediction of Ohtsuki and Keyes.

Diffusion in disordered media has been studied when a constant force is applied to the random walker, pushing it in one direction (Böttger and Bryskin 1982, Ohtsuki 1982, Barma and Dhar 1983, Ohtsuki and Keyes 1984, Dhar 1984, White and Barma 1984). Monte Carlo studies (Pandey 1984, Seifert and Suessenbach 1984) showed very complex behaviour for concentrations p of allowed sites greater than the percolation threshold p_c. The present study concentrates on this biased diffusion right at $p = p_c$, for two and three dimensions, in the hope that the behaviour is simpler here.

Dhar predicts $k(p = p_c) = 0$ for the asymptotic critical behaviour defined through

$$R \propto t^k \qquad (t \to \infty) \tag{1}$$

where R is the RMS distance from the origin of the walk, and t the time, in units of the lattice constant and the inverse jump rates, respectively; $1/k$ may also be denoted as the fractal dimension d_w of that walk. The preliminary data published by Pandey (1984) at $p = p_c$ already show that most probably the distance R does not approach a constant (as it does below p_c), and suggests a small k or logarithmic variation:

$$R \propto (\log t)^x \tag{2}$$

with some unknown exponent x. Such a behaviour, too, would be consistent with Dhar's prediction $k = 0$. We test the validity of equations (1) and (2) by using the methods of Pandey (1984) in more detailed runs on the CDC Cyber 205 vector computer (Seifert and Suessenbach 1984). However, we always took the force and one lattice direction (triangular or simple cubic lattice) in the x direction. We found no significant changes in the results if we used a Tausworth shift generator (Kalle and Wansleben 1984) instead of the random number generator RANF.

From our data we determine the effective exponent

$$k(t) = \mathrm{d}\ln r/\mathrm{d}\ln t \tag{3}$$

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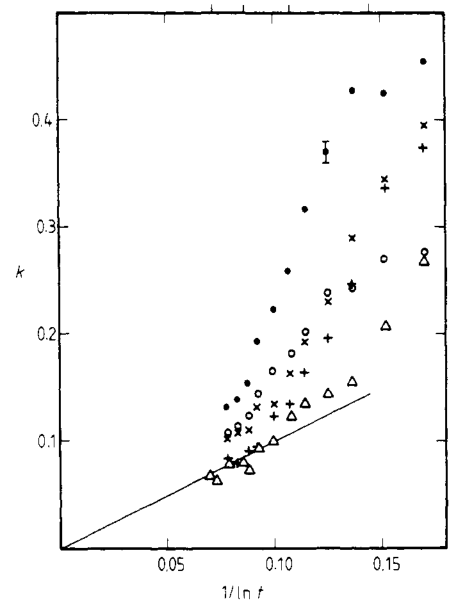

Figure 1. Effective exponent $k(t)$ against $1/\ln(t)$, for triangular and simple cubic lattices. The sizes and bias force B are, respectively, ●, 4096^2, 0.2; ×, 4096^2, 0.5; +, 4096^2, 0.8; ○, 176^3, 0.2; △, 256^3, 0.8 m. The straight line corresponds to an asymptotic behaviour as in equation (2) with $x = 1$.

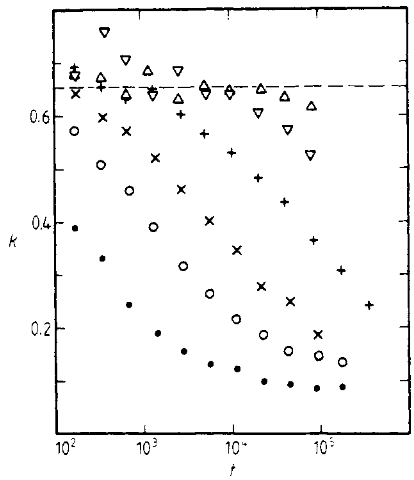

Figure 2. Effective exponent k against $\ln(t)$ for the average displacement $\langle x \rangle$ in the direction of the force B, as follows: △, 0.01; ▽, 0.02; +, 0.04; ×, 0.1; ○, 0.2; ●, 0.8; the symbol size is of the order of the statistical errors. The broken line indicates the height of the plateau for small forces B and intermediate times t in these triangular lattices (2560^2 and 4096^2 sites, 6 to 40 runs with 512 ants each; for $B = 0.01$ we made 250 runs). Three-dimensional results are similar but less accurate, with a plateau near 0.4.

which is plotted in figure 1 against $1/\ln t$. The force is measured dimensionlessly through the bias field B: with probability $(1+5B)/6$ the walker selects the x direction and with probability $(1-B)/6$ any other direction for its next attempt to move. Thus $B=1$ corresponds to infinite forces and $B=0$ to zero force. For both strong and weak bias fields B, the effective exponent decays with increasing time, in both two and three dimensions. Figure 1 suggests strongly that the asymptotic $k(t \to \infty)$ is not appreciably positive. Since k cannot be negative it is presumably zero, though a value like $k=0.03$ just seems possible. Assuming k to vanish asymptotically, we find the exponent x in equation (2) as the slope of the straight line through the data of figure 1. Our data are consistent with x of the order of unity in three dimensions, and perhaps somewhat higher in two. The corresponding fractal dimension of this walk is infinite, i.e. time increases exponentially with distance.

This logarithmic behaviour is not necessarily in contradiction with the power law predicted by Ohtsuki and Keyes (1984). From their work we expect at intermediate times a power law for the mean displacement $\langle x \rangle \propto t^k$ in the direction of the force for times smaller than some negative power of B but much larger than unity. Indeed, for the required small B our data for $\langle x \rangle$ in figure 2 suggest a plateau in $k(t)$ for intermediate times, near $k=0.66$ in two dimensions and 0.42 in three dimensions. These values are close to those for the mean square distance in zero field, as predicted (T Ohtsuki, private communication). Note that our simulations average over all clusters, finite and infinite. (For another theory see Kholodenko (1985).)

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